

Math 62: 8.3 Piecewise Functions
Lesson #2

Math 72: 3.4 Piecewise Functions
Lesson # 11-12

3.8 - 2nd Piecewise Functions

Objectives: 1) Graph piecewise-defined functions.

To do this we need several smaller objectives:

- a) Understand notation used for piecewise functions.
- b) Evaluate piecewise functions
- c) Determine y-values at endpoints of piecewise functions.
- d) Determine if endpoints should be graphed with $\bullet$ or $\circ$.

GC 21: Graphing Piecewise Functions

An example of a piecewise-defined function, also called a piecewise function:

$$f(x) = \begin{cases} 2x+3 & \text{if } x \leq 0 \\ -x-1 & \text{if } x > 0 \end{cases}$$

Note: The word "if" is optional

$f(x)$ is the name of this function.

The brace $\{$ tells us it's piecewise.

$(2x+3)$ and $(-x-1)$ are the two pieces.

$x \leq 0$ is the domain for the first piece $(2x+3)$.

$x > 0$ is the domain for the second piece $(-x-1)$.

Note: It has one name because it's all one function.

The resulting graph

MUST PASS THE VERTICAL LINE TEST!

① Using $f(x)$ above, evaluate.

a) $f(3)$

b) $f(1)$

c) $f(0)$

d) $f(-1)$

① continued.

a) $f(3)$

step 1: Identify which domain is true for this x .
 $x=3$ is greater than 0 $\Rightarrow x > 0$.

step 2: Use the piece which applies to that domain.
(Ignore the other piece!)

$$-x - 1 \text{ if } x > 0.$$

step 3: Evaluate the appropriate piece.

$$\begin{aligned} & -x - 1 \\ & = -(3) - 1 \\ & = -4 \end{aligned}$$

$$\boxed{f(3) = -4}$$

b) $f(1)$

step 1: (as above) $x=1$ is $x > 0$.

step 2: $-x - 1$ if $x > 0$

step 3: $-(1) - 1$

$$\boxed{f(1) = -2}$$

c) $f(0)$

step 1: $x=0$ is $x \leq 0$

step 2: $2x + 3$ if $x \leq 0$

step 3: $2(0) + 3$

$$\boxed{f(0) = 3}$$

d) $f(-1)$

step 1: $x=-1$ is $x \leq 0$

step 2: $2x + 3$ if $x \leq 0$

step 3: $2(-1) + 3$

$$\boxed{f(-1) = 3}$$

② using the same $f(x)$, evaluate to complete the tables.

Hint: Use GC table function.

x	$f(x)$
-5	
-4	
-3	
-2	
-1	
0	

x	$f(x)$
3	
2	
1	
$\frac{1}{2}$	
$\frac{1}{4}$	
$\frac{1}{8}$	

Solution:

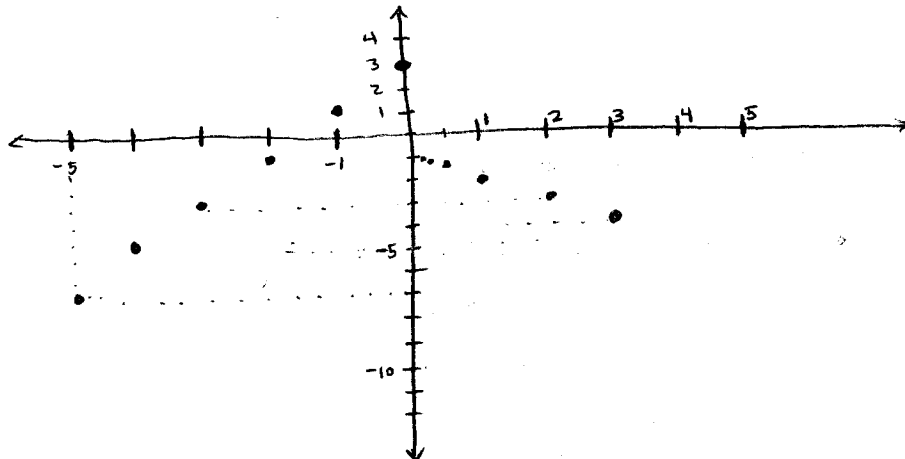
x	$f(x)$
-5	-7
-4	-5
-3	-3
-2	-1
-1	1
0	3

use $2x+3$
piece

x	$f(x)$
3	-4
2	-3
1	-2
$\frac{1}{2}$	-1.5
$\frac{1}{4}$	-1.25
$\frac{1}{8}$	-1.125

use $-x-1$
piece

③ Sketch the graph of the points we know so far.



④ Complete the graph by

- calculating endpoints of each piece
- using $\bullet$ to graph piece where endpoint is included ($x \leq 0$ includes $x=0$).
- using $\circ$ to graph piece where endpoint is not included ($x > 0$ does not include $x=0$).
- connecting dots within each piece

The endpoints occur when $x=0$, from domains.

$f(0) = 3$ is the $\bullet$ endpoint of the piece $2x+3$.

The endpoint of the piece $-x-1$ is also when $x=0$.

But this endpoint is not included.

Substitute $x=0$ to get y -coordinate.

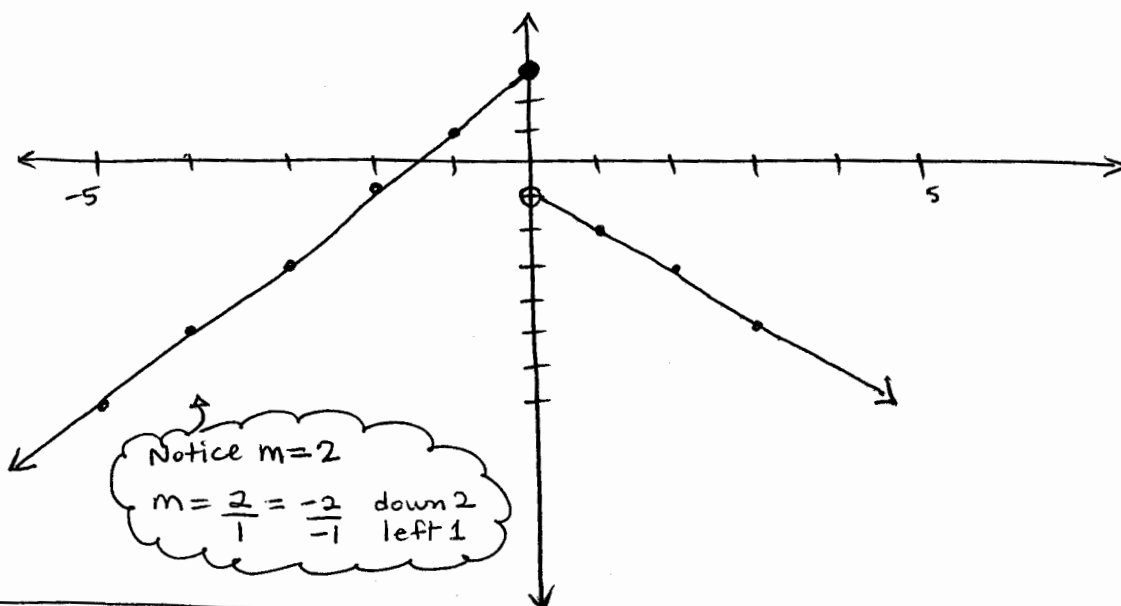
$$-0-1 = -1$$

Graph $(0, -1)$ using open circle $\circ$.

Connect all the $x \leq 0$ pts together.

Connect all the $x > 0$ pts together.

Note: The left piece may or may not connect to the right piece.



Note: Do NOT put arrow heads at the endpoints where domains are divided.

Note: Do put arrowheads on the ends where $x \rightarrow +\infty$ or $x \rightarrow -\infty$.

m70

⑤ Sketch graph of $f(x) = \begin{cases} -2x+4 & x \leq -1 \\ 3 & x > -1 \end{cases}$

step 1: $-2x+4$ is a line with slope -2
 3 is a horizontal line

step 2: Endpoints occur when $x = -1$

$$f(-1) = -2(-1) + 4 = 6$$

plot $(-1, 6)$ using $\bullet$ included endpt.

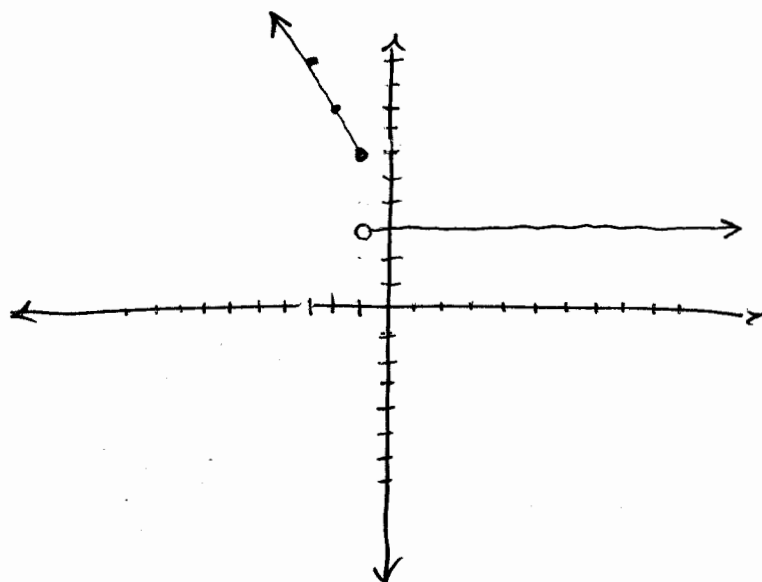
$3 \Rightarrow$ plot $(-1, 3)$ using $\circ$ excluded endpt.

step 3: To graph $-2x+4$, $x \leq -1$

plot $(-1, 6)$, use $m = \frac{-2}{1} = \frac{2}{-1} = \begin{matrix} 2 \text{ up} \\ 1 \text{ left} \end{matrix}$
 using $\bullet$

To graph 3 , $x > -1$

plot $(-1, 3)$ using $\circ$, draw horizontal.



Yes, it passes the V.L.T.!

Helpful Hint: For the $x < \#$ or $x \leq \#$ piece, write slope as fraction with negative denominator. (go left).

For the $x > \#$ or $x \geq \#$ piece, write slope as fraction with positive denominator (go right)

Procedure for graphing piecewise functions with linear pieces.

$$f(x) = \begin{cases} ax + b & x \leq c \\ dx + e & x > c \end{cases}$$

- 1) draw axes.
- 2) identify $x=c$ on graph, left & right pieces
 $x \leq c$ $x > c$
(or $x \leq c$ & $x \geq c$)
- 3) Subst $x=c$ into 1st piece
use $\bullet$ if $x \leq c$ or $x \geq c$
use $\circ$ if $x < c$ or $x > c$
- 4) use slope of first piece
to fill in line in correct direction
- check domain
- check sign of slope
- 5) repeat steps 3 & 4 for 2nd piece.
- 6) check that final graph passes VLT.

Now that we have explored a problem in detail, let's write our method.

step 1: Identify the shape of each piece. [In M70, usually lines.]

step 2: Evaluate endpoints in both pieces.
Determine which is $\bullet$ and which is $\circ$ on graph.

step 3: Graph each piece.

step 4: Check that final graph passes V.L.T.

⑤ Sketch graph of $f(x) = \begin{cases} x+2 & \text{if } x < 1 \\ 2x+1 & \text{if } x \geq 1 \end{cases}$

step 1: $x+2$ is a line $y=x+2$ with slope 1.
 $2x+1$ is a line $y=2x+1$ with slope 2.

step 2: Endpoints occur when $x=1$.

$$f(1) = 2(1) + 1 = 3$$

plot $(1, 3)$ using $\bullet$ included endpoint

In other piece $1+2=3$

plot $(1, 3)$ using $\circ$ excluded endpoint

Note: In this graph, the endpoints overlap!

step 3: To graph $x+2$, $x < 1$

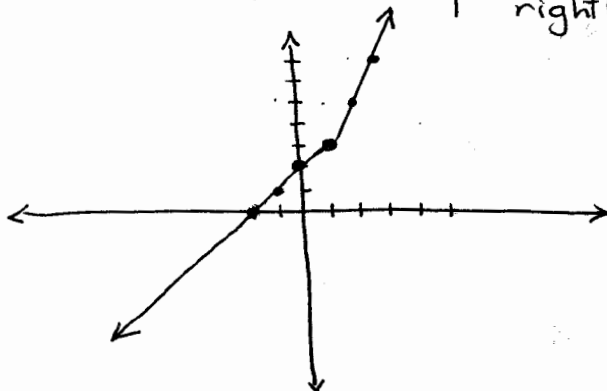
plot $(1, 3)$... with $\circ$ excluded endpoint

use slope $m = \frac{1}{1} = \frac{-1}{-1}$ down 1, left 1

To graph $2x+1$, $x \geq 1$

Plot $(1, 3)$ with $\bullet$ included endpoint

use slope $m = \frac{2}{1}$ up 2, right 1



Yes! It passes the VLT!

Extra Practice: Sketch graphs

$$\textcircled{7} \quad f(x) = \begin{cases} x-1 & x \leq 3 \\ -x+5 & x > 3 \end{cases}$$

$$\textcircled{8} \quad f(x) = \begin{cases} 4x-4 & x < 2 \\ -x+1 & x \geq 2 \end{cases}$$

$$\textcircled{9} \quad g(x) = \begin{cases} -3x & x \leq -2 \\ 3x+2 & x > -2 \end{cases}$$

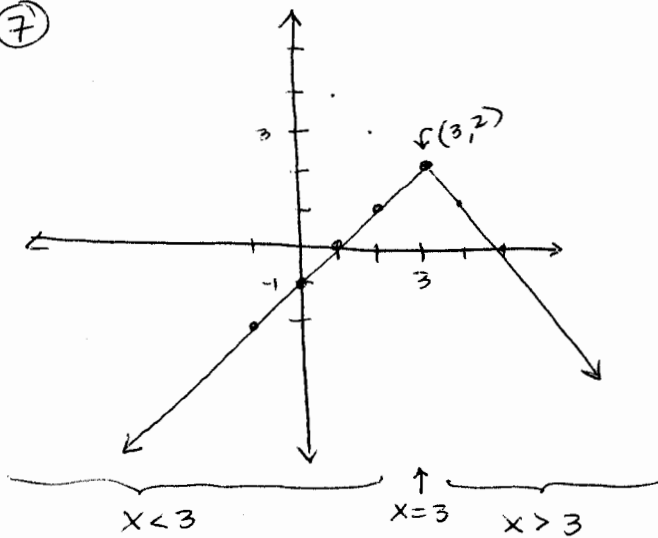
$$\textcircled{10} \quad k(x) = \begin{cases} -1 & x < -3 \\ -2 & x \geq -3 \end{cases}$$

$\textcircled{11}$ Challenge question

$$m(x) = \begin{cases} -x+2 & x < -1 \\ -x^2+4 & x \geq -1 \end{cases}$$

Solutions to Extra Practice

⑦



Note:

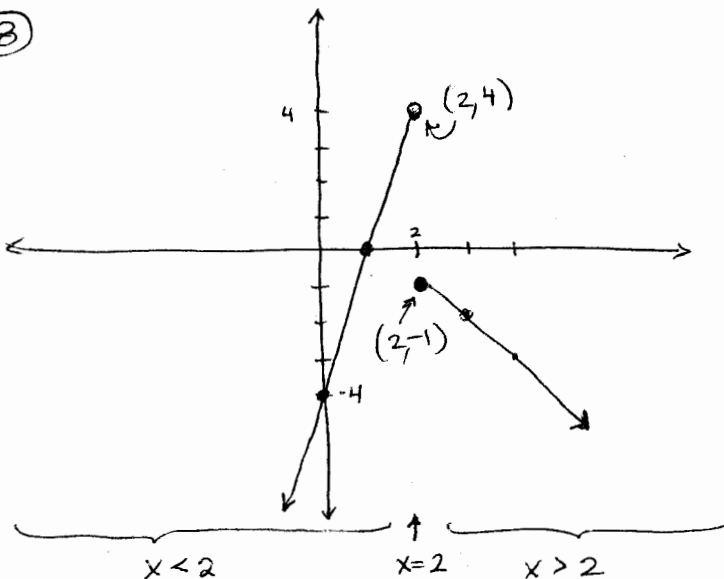
$$f(x) = \begin{cases} x-1 & x \leq 3 \\ -x+5 & x > 3 \end{cases}$$

is the same as

$$f(x) = -|x-3| + 2.$$

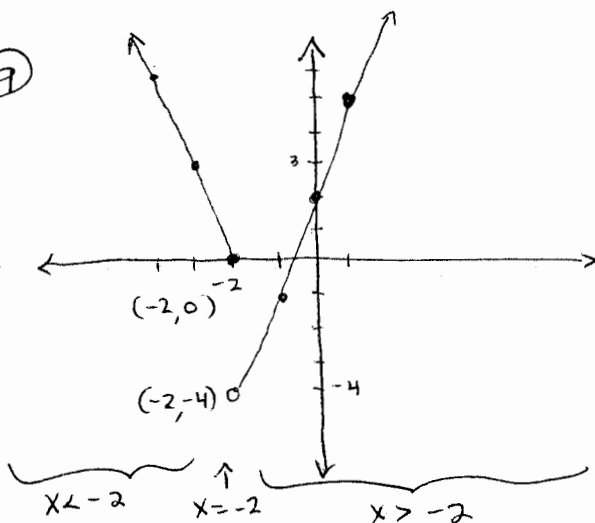
ALL absolute functions can be written as piecewise functions.

⑧



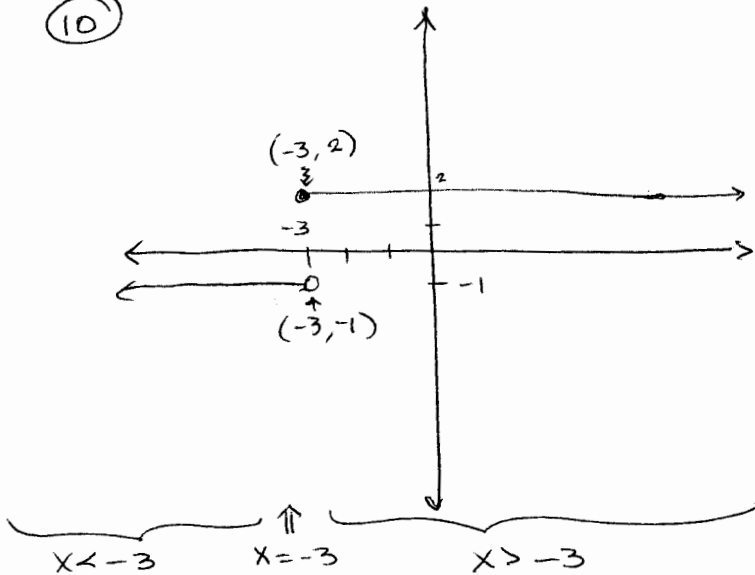
$$f(x) = \begin{cases} 4x-4 & x < 2 \\ -x+1 & x \geq 2 \end{cases}$$

⑨



$$g(x) = \begin{cases} -3x & x \leq -2 \\ 3x+2 & x > -2 \end{cases}$$

(10)

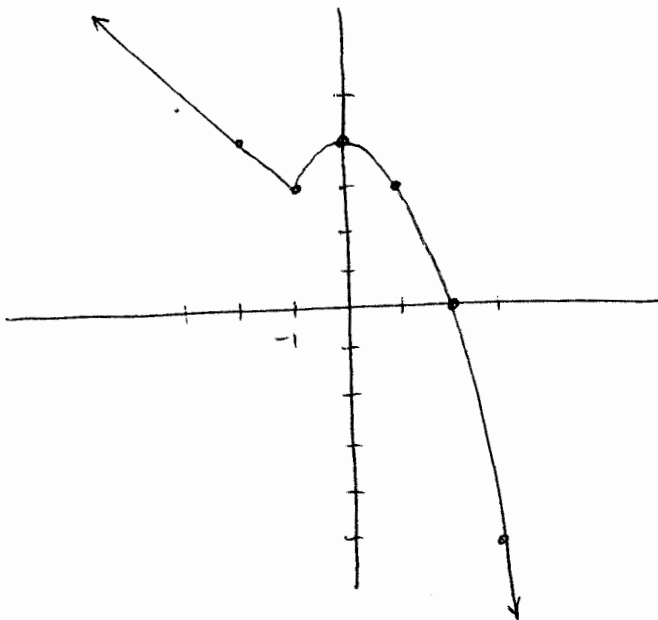


$$k(x) = \begin{cases} -1 & x < -3 \\ 2 & x \geq -3 \end{cases}$$

(11)

$$m(x) = \begin{cases} -x+2 & x < -1 \\ -x^2+4 & x \geq -1 \end{cases}$$

lines

parabola opening down
vertex at (0,4)

$$m(-1) = -(-1)^2 + 4 = 3$$

(-1, 3) plotted ●

$$\text{other piece } -(-1) + 2 = 3$$

they connect at (-1, 3).

Name _____

Date _____

TI-84+ GC 21 Graphing Piecewise Functions

- Objectives:
- Use TEST to compare a single value of a variable to a test value
 - Use TEST to compare many values of a variable to a test value
 - Multiply a function by a TEST result to limit domain and graph a piece
 - Determine if your GC will show $y = 0$ when multiplied, use divide by a TEST result
 - Add pieces to graph a piecewise function

The TEST menu is the second function above MATH:

2ND

MATH

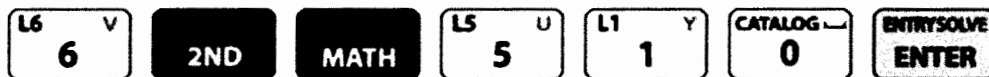
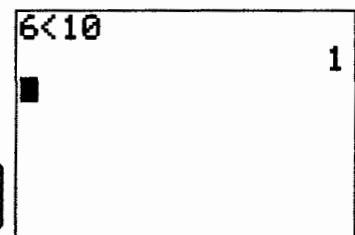


We can select from the list by typing the corresponding number. "Less than" is option 5, for example.

Example 1: Use GC to determine if

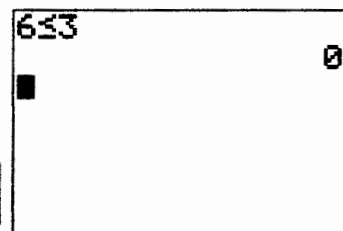
- a) $6 < 10$
- b) $6 \leq 3$

a) Use the TEST menu to determine if $6 < 10$



The GC says 1 if the answer to the question is true (or yes).
 $6 < 10$ is a true statement, so GC says 1.

b) Use the TEST menu to determine if $6 \geq 3$

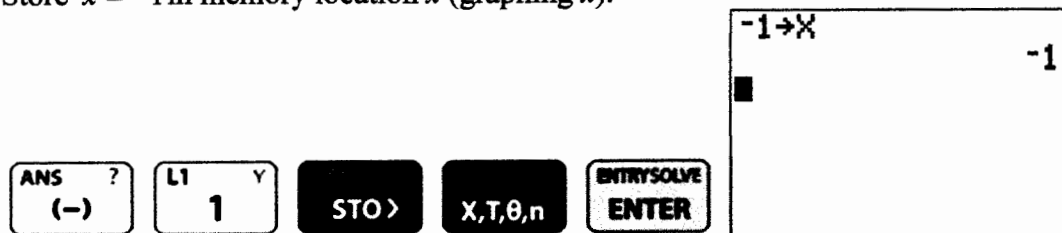


The GC says 0 if the answer to the question is false (or no).
 $6 \geq 3$ is a false statement, so GC says 0.

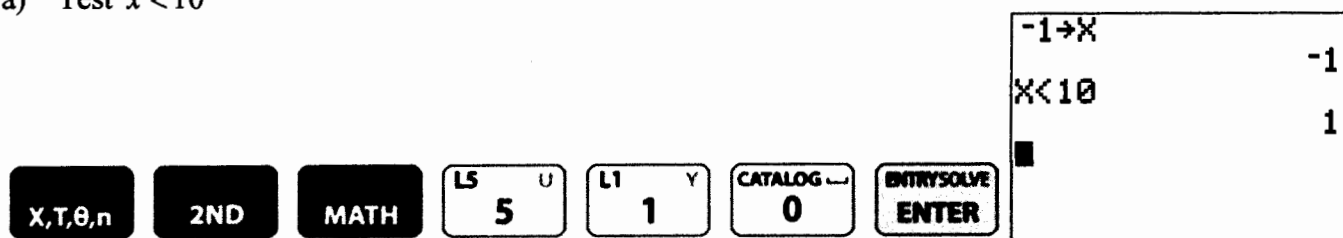
Example 2: Use GC to store $x = -1$ in memory and determine if

- a) $x < 10$
b) $x \geq 3$

Store $x = -1$ in memory location x (graphing x).

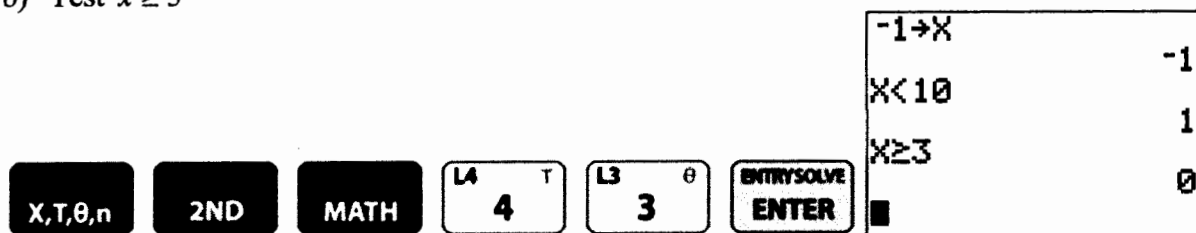


a) Test $x < 10$



The GC tests the value stored in memory, and concludes that yes, $-1 < 10$.

b) Test $x \geq 3$



The GC tests the value stored in memory, and concludes that no, $-1 \geq 3$ is a false statement.

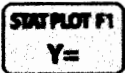
Example 3: Graph the TEST results for

- a) $x \leq 4$
b) $x > -2$

CAUTION: We are NOT graphing the inequality $x \leq 4$, which would be graphed on a number line.

TEST gives y-coordinate that is 0 or 1, depending on whether x is less than or equal to 4 (y-coordinate 1) or not less than or equal to 4 (y-coordinate 0).

The GC automatically generates the values of x from Xmin and Xmax in WINDOW.

a) Type the test $x \leq 4$ as a function in the  menu, then graph in a standard viewing window.

Example 3, continued

STAT PLOT F1 Y= X,T,θ,n 2ND MATH L6 V L4 T 6 4

FORMAT F3 ZOOM L6 V 6

Plot1 Plot2 Plot3
Y1 $X \leq 4$
Y2 =
Y3 =
Y4 =
Y5 =
Y6 =
Y7 =

The GC graphs a horizontal line $y = 1$ for all x values up to $x = 4$, then a horizontal line $y = 0$ for all x -values after 4. It's hard to tell what it did at exactly $x = 4$ without checking a table.

2ND TABLE F3 GRAPH X=6

X	Y1
0	1
1	1
2	1
3	1
4	1
5	0
6	0

$x = 4$ has y -value 1.

b) Type the test $x > -2$ as a function in the STAT PLOT F1 Y= menu, then graph in a standard viewing window.

STAT PLOT F1 Y= X,T,θ,n 2ND MATH L3 θ L2 Z 3 (-) 2

FORMAT F3 ZOOM L6 V 6

Plot1 Plot2 Plot3
Y1 $X > -2$
Y2 =
Y3 =
Y4 =
Y5 =
Y6 =
Y7 =

The GC graphs a horizontal line $y = 0$ for all x values before $x = -2$, then a horizontal line $y = 1$ for all x -

values after -2.

X	Y1
-3	0
-2	0
-1	1
0	1
1	1
2	1
3	1
4	1

X=-2

The table shows that $x = -2$ is assigned y -coordinate 0.

Example 4: Graph the function $f(x) = 2x + 3$ for the domain $x \leq 0$ by multiplying by the TEST results for its domain.

CAUTION: We want the function value times the test value, so we need to include additional parentheses!

Keystrokes for Example 4:

STAT PLOT F1 Y= (L2 2 X,T,θ,n MEM + L3 3) [X R (

X,T,θ,n 2ND MATH L6 6 V CATALOG 0) ENTRY/SOLVE ENTER

FORMAT F3 ZOOM L6 6 V

Calculator screen showing the function definition:

```

Plot1 Plot2 Plot3
Y1=(2X+3)*(X≤0)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

```

CAUTION: If you have a new color calculator, you may see a horizontal line $y = 0$ for the x -values on the right side of the graph. You can remove this by *DIVIDING* by the test value. Newer calculators do not graph values that result in $\div 0$.

Keystrokes and screenshot for dividing by the test value:

Keystrokes for dividing by the test value:

STAT PLOT F1 Y= (L2 2 X,T,θ,n MEM + L3 3) ÷ (

X,T,θ,n 2ND MATH L6 6 V CATALOG 0) ENTRY/SOLVE ENTER

Calculator screen showing the function definition:

```

Plot1 Plot2 Plot3
Y1=(2X+3)/(X≤0)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

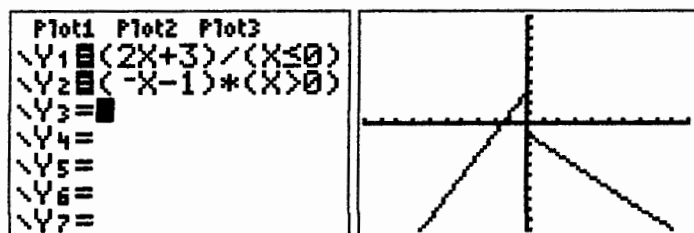
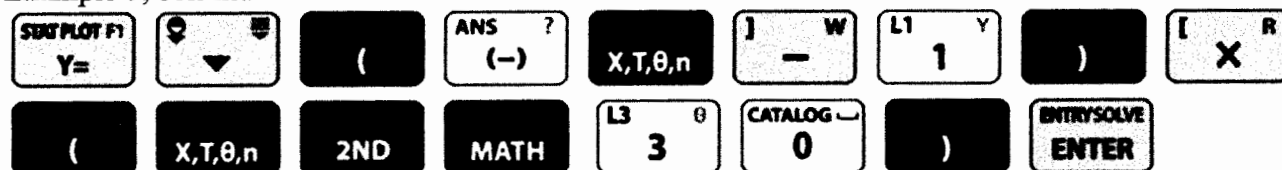
```

CAUTION: However, if you have an older calculator, dividing by the test value will completely confuse your calculator and it may graph nothing at all!

Example 5: Graph the piecewise function $f(x) = \begin{cases} 2x + 3 & x \leq 0 \\ -x - 1 & x > 0 \end{cases}$ by multiplying each piece by TEST results for its domain using two lines of the $y=$ menu.

The first piece is the same as Example 4. We continue with the second piece in y_2 :

Example 5, continued

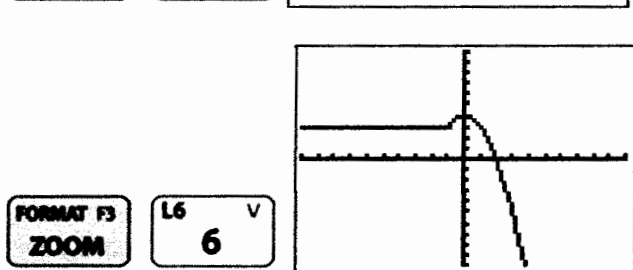
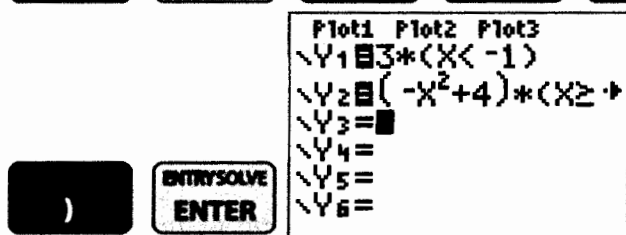
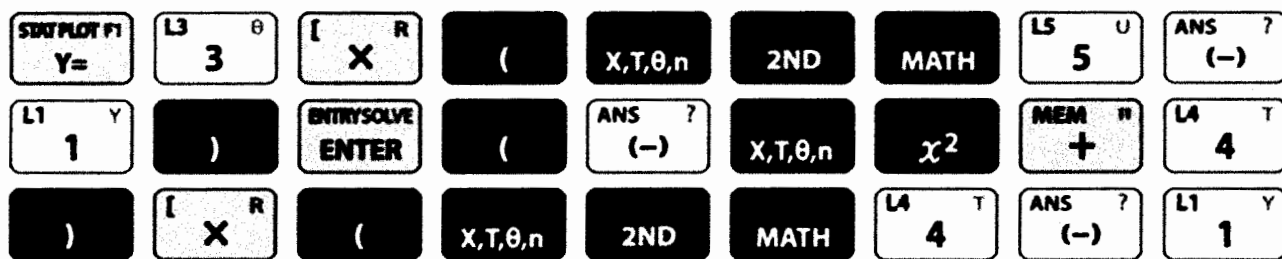


CAUTION: When you transfer this graph to paper, you must clearly indicate which piece includes its endpoint (closed circle) and which piece does not (open circle)!

REMEMBER: A piecewise function is still a single function, and the resulting graph must pass the vertical line test!

Example 6: Graph the piecewise function $f(x) = \begin{cases} 3 & x < -1 \\ -x^2 + 4 & x \geq -1 \end{cases}$.

We expect the first piece to be a horizontal line while the second piece is a parabola.



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Practice: Graph the piecewise function.

1) Graph $f(x) = \begin{cases} x+2 & x < 1 \\ 2x+1 & x \geq 1 \end{cases}$

2) Graph $f(x) = \begin{cases} -2x+4 & x \leq -1 \\ 3 & x > -1 \end{cases}$

3) Graph $f(x) = \begin{cases} x-1 & x \leq 3 \\ -x+5 & x > 3 \end{cases}$

4) Graph $f(x) = \begin{cases} 4x-4 & x < 2 \\ -x+1 & x \geq 2 \end{cases}$

5) Graph $f(x) = \begin{cases} -3x & x \leq -2 \\ 3x+2 & x > -2 \end{cases}$

6) Graph $f(x) = \begin{cases} -1 & x < -3 \\ 2 & x \geq -3 \end{cases}$

7) Graph $f(x) = \begin{cases} -x+7 & x < -1 \\ -x^2+9 & x \geq -1 \end{cases}$

